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Modelled Energy Cost Minimization Solution for Wireless Rechargeable Sensor Networks

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Abstract: *In wireless rechargeable sensor networks (WRSNs), mobile chargers (MCs) are normally scheduled to deliver energy to the rechargeable sensor nodes (SNs). However, due to the energy consumption dynamicity of WRSNs, constructing optimal charging trajectories with minimized number of failed SNs due to energy deficiency ensuring a sustained WRSN operation at minimum MC's movement cost is one aspect of the subject matter not yet thoroughly investigated. Thus, exploring this knowledge is the focus of this work. We applied shortest path algorithm, on-demand scheduling and multi-node charging methods to construct the energy cost-effective charging path for the MC, a model we coined as Shortest Hamiltonian Cycle Traveling Salesman Problem (SHC-TSP). Comparative analysis proves the optimality of our solution against the notable nearest job next with pre-emption (NJNP) model in terms of minimizing MC's traveling energy cost with energy savings of 3.9156% and 2.1940% for the two scenarios respectively examined.*

Keywords: *Mobile Charger, On-demand Charging Schemes, Sensor Nodes, Shortest Path Algorithm, Wireless Rechargeable Sensor Networks*

1. INTRODUCTION

Sensor nodes (SNs) in wireless sensor networks (WSNs) are normally powered with small batteries of limited capacities and in many application scenarios are deployed into a risky, remote or inaccessible environments for long-term operation, making replacement of the batteries highly laborious and costly [1]. Obviously, energy is a very precious but scarce resource in WSNs whose improvement and efficient utilization is currently a research hub in WSNs domain [2]. Credence to the recent technological advancements in wireless power transfer (WPT), a new solution using wireless mobile charger (MC) called wireless rechargeable sensor network (WRSN) emerged [3].

The drawbacks of previous charging solutions which are mostly based on periodical and overall charging [4, 5] drive research for a more energy-efficient solution called *on-demand* [6]. In on-demand charging, the MC is schedule to locate and recharge only those SNs that used their energy below the threshold and send a request for recharging. Hence, it is more flexible, adaptive and energy-efficient [7]. However, due to the heterogeneity of WSNs' energy consumption profiles and the corresponding finite MCs' utility energies, planning an on-demand charging trajectory to mitigate the number of failed SNs due to battery energy depletion ensuring a sustained network operation at minimum MC's moving energy cost is still a challenging research problem [8, 9] which this work investigates.

In a similar approach Zhao *et al.* [10] and Liu *et al.* [11] investigate charging scheduling problem to mitigate sensing holes and sustain the network's connectivity based on partial charging. However, this approach requires the MC to travel continuously nonstop to meet the objective; thus, results to high MC's energy consumption. Wang *et al.* [12] and Tian *et al.* [13] develop recharging models primarily to minimize nodes failure. But, to simplify their solution, they failed to consider the energy consumed by the MCs which contradicts real-life application. In their work, Wang *et al.* [14] solves the charging problem to reduce the network operating cost by minimizing the required number of MCs using NJNP approach. However, just like shortest path, NJNP scheduling increases the waiting time of faraway SNs which consequently leads to their premature death but presents a higher MC's travelling cost as compared to the shortest path. The methodology applied in [15] to address similar problem, assumed that the MC has sufficient energy to charge all the SNs in every cycle while that applied in [16] divided the SNs into three categories (a, b and c) based on their residual energy levels and optimize their charging to conserve energy. However, these assumptions are too ideal and unrealistic. In contrast, our assumption here is that the MC has limited energy that can be exhausted at any time, depending on the workload.

2. THEORETICAL FRAMEWORK

This section defines the charging model and the relevant performance evaluation metrics being the method and parameters used to model and validate the solution, respectively.

2.1 Charging Model

The charging model used in this work is the radio model [17] given by Equation (1):

$$P_r(s_i, MC) = \frac{G_t G_r \eta}{L_p}\left(\frac{\lambda}{4\pi(dist(s_i,MC) + \delta)}\right)^2 P_t \tag{1}$$

Where; P_t is the transmit power of the MC, P_r is the received power of the SN, $dist(s_i, MC)$ is the Euclidian distance between the SN s_i and the MC during charging, G_t is the MC antenna gain, G_r is the SN antenna gain, L_p is the antenna polarization loss $(0 \leq L_p \leq 1)$, λ is the RF radiation wavelength, η is the rectifier efficiency and δ is the parameter to adjust the Friis' free space equation for short distance radio links whose value is 0.2316.

2.2 Energy Utility $(\mathbf{E_u})$

This is the MC's energy efficiency index, defined as the ratio of the energy it used to recharged the SNs (payload energy, E^{PL}) to the energy it consumed for movement (overhead energy, E^{OH}) in each charging round, neglecting any energy loss [6, 18] as expressed in Equation (2):

$$E_u = \frac{E^{PL}}{E^{OH}} = \frac{\sum_{i=1}^{w} q_c\, C_t\, C_e}{q_m\, D_p} \tag{2}$$

Where; q_c is the charging power of the MC, C_t is the duration that the MC charges the ith SN s_i, C_e is the charging efficiency of the MC, q_m is the MC's moving energy consumption, D_p is the total distance traveled by the MC and w is the number of SNs in the charging schedule.

2.3 Charging Time $(\mathbf{C_t})$

This is the time taken by the MC to charge a SN s_i [19] as presented in Equation (3):

$$C_t = \frac{E_{max} - E_{min}}{P_r} \tag{3}$$

Where; P_r is the power received by the SN s_i while E_{max} and E_{min} are the initial and residual energies of the SN s_i being charged.

Hence, higher value of utility (E_u) indicates that the MC has utilized more of its energy in charging the SNs than to travel; thus, indicating improved energy efficiency for the MC.

3. MATERIALS AND METHODS

3.1 Materials

To achieve the objectives of the research, Matlab R2022b simulation software was deployed. Internet, and online journals, textbooks and thesis were also consulted.

3.2 Methods

This section discusses the methods adopted to model and simulate the system.

3.2.1 Network modelling and charging behaviour: As a strategy to achieve the research goal, the multi-node charging approach which enables the MC to simultaneously charge multiple SNs within its charging range is found most appropriate. Thus, a state-of-the-art multi-node charging scheduling scenario which requires partitioning the two-dimensional (2-D) network region into a number of regular hexagonal cells is explored. In this method, the centre of each cell is considered as the docking spot [20]. This new approach enables the transformation of traveling path optimization problem into a charging scheduling problem; thus, saves waiting time, movement cost, and indeed improves the scalability of the recharging scheme [21]. The network model for a particular scenario is shown in Figure 1.

Let the network consists of a set of SNs $S = \{s_1, s_2, s_3 \dots\dots s_i, \dots, s_n\}, i \in [1, n], i \in \mathbb{N}^+$, where $\mathbb{N}^+$ represent a set of positive integers and s_i is the i^{th} SN distributed over the network area. The coordinates of each SN $s_i \in S$ is (x_i, y_i). It is assumed that all the SNs are identical with same computing and communication capabilities. Also, each SN s_i is equipped with the same rechargeable battery of capacity E_{max}. The minimum energy level for normal working of SN s_i is E_{min}, which is the energy threshold for a SN s_i to send a charging request.

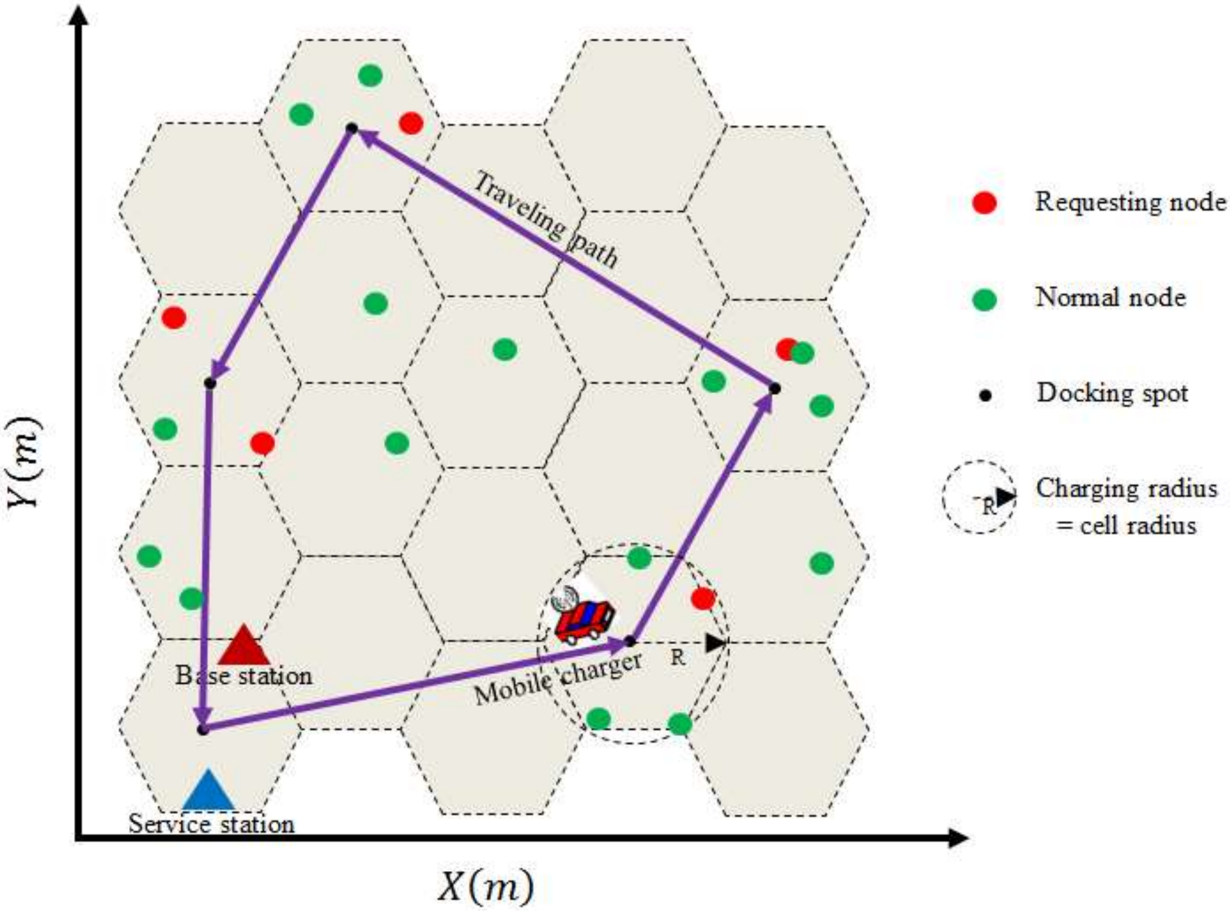

Figure 1: Network model based on hexagonal cells structure for a particular scenario

Notice that the radius of the cells is initially adjusted to be equal to the charging radius of the MC (which is bearing RF power-caster with Omni-directional antenna). After serving the request of a cell, the MC moves to the next cell along its charging path, which is based on shortest Hamiltonian cycle that solves the traveling salesman problem (TSP) (optimal path). Note that the MC can follow clockwise or anti-clockwise direction to achieve the same path length. After serving the entire requests the MC will return to its base, recharge its battery and wait for the next round. This charging process is summarized in the flowchart shown in Figure 2.

Under the cellular structure, we denote $d_{i,k}$ as the Euclidean distance from SN s_i and its cell center k while $D_{i,j}$ denote the Euclidean distance between the cell centres of two cells containing requesting SNs along the Hamiltonian path, respectively represented by Equations (4) and (5). But, it should be noted that the distances between the SNs and the cell centre are different. Therefore, since the MCs' charging power fades over its charging distance, the charging efficiency degrades over the charging distance too. Thus, each SN receives charging energy proportional to its location relative to its cell centre. Therefore;

$$d_{i,k} = \sqrt{(x_i - X_k)^2 + (y_i - Y_k)^2} \ , i \,\&\, k \in \mathbb{N}^+, i \neq k \tag{4}$$

$$D_{i,j} = \sqrt{\left(X_i - X_j\right)^2 + \left(Y_i - Y_j\right)^2} \ , i \,\&\, j \in \mathbb{N}^+, i \neq j \tag{5}$$

The number of SNs in a cell is a random variable. Some SNs whose battery energy lessens below E_{min} will send a charging request to the base station (BS). Thus, a cell may have N number of charging request. Hence, $0 \leq N \leq \rho$, where ρ is the maximum number of SNs in i^{th} cell.

3.2.2 Charging planning and cycle time analysis: Let Q denote the set of cells containing at least one SN with charging request. Also, let τ_k represent the time the MC stays at the center of the cell to simultaneously charge all the SNs within. After τ_k the MC leaves the current cell and moves to the next cell along its charging path. In this solution, it is assumed that the MC visit a cell only once during a cycle. Let P represent the path traversed by the MC during a cycle, which begins from and ends at the SS. Thus, P may be represented by the relation in Equation (6):

$$P = (c_0, c_{1,} \dots \dots, c_k, \dots \dots c_{|Q|}, c_0),\ k \in [1, |Q|],\ k \in \mathbb{N}^+ \tag{6}$$

Where; c_0 and c_k represent service station (SS) and kth cell centre visited by the MC, respectively. Now, let the distance between two neighbouring cell centres along the charging path be $D_{c_{k,}c_{k+1}}$, then $D_{c_{0,}c_k}$ or $D_{c_{k,}c_0}$ is the distance between SS and its neighbouring cell centre.

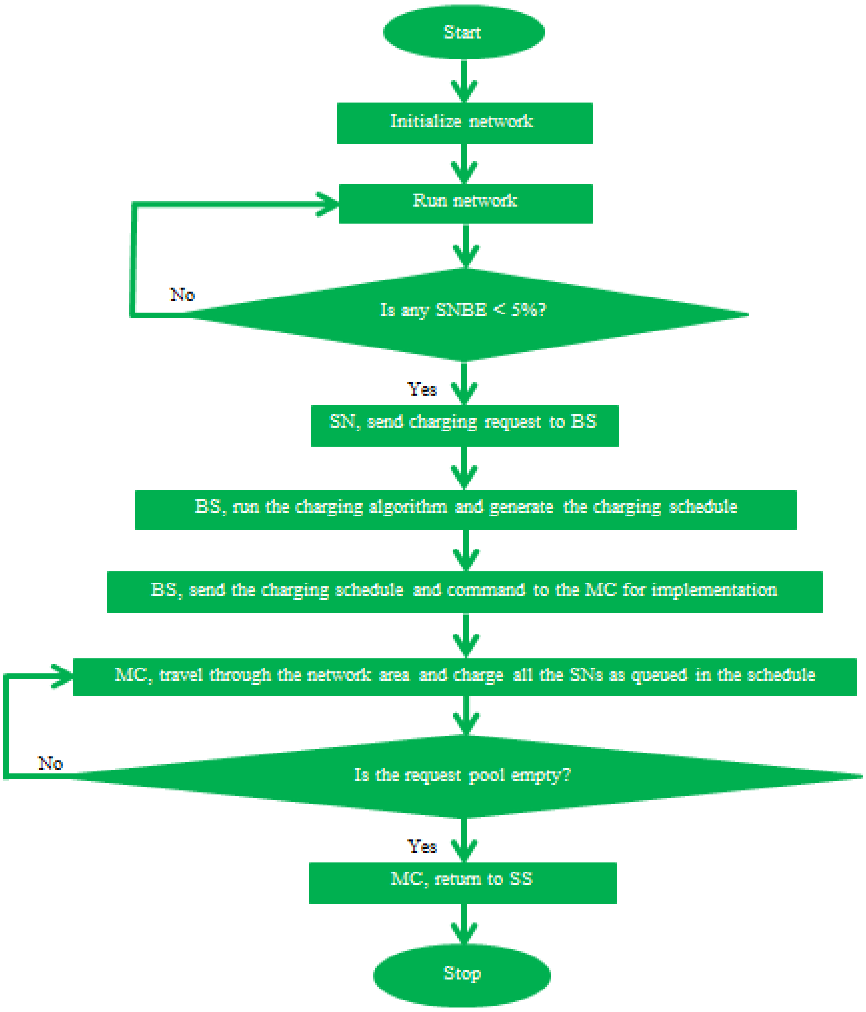

Figure 2: Flowchart of the charging process

The working state of the MC is divided into three: moving, charging and vacation. Thus, the moving time τ_p of MC should satisfy the condition in Equation (7):

$$\tau_p = \sum_{k=0}^{|Q|-1} \frac{D_{c_{k,}c_{k+1}}}{v} + \frac{D_{c_{|Q|-1,}c_0}}{v} \tag{7}$$

The k^{th} cell centers traversed by the MC along the charging path is $c_k, 1 \leq k \leq |Q|$. Also, denote D_p as the distance of charging path; therefore, $\tau_p = D_p/v$ is the time spent for traveling over the distance D_p.

In the charging state, the MC stays at the cell centre and charges all the SNs in the cell including the normal ones (those that are yet to requested for recharging). Thus, the charging time in the k^{th} cell is denoted as τ_k. Then, the total charging time τ_c should satisfy the condition in Equation (8):

$$\tau_c = \sum_{k \in Q} \tau_k \tag{8}$$

After the MC finished visiting the Q cells, it will return to its SS to be serviced (recharge or replace its battery) and rest before the next tour. This resting period is usually called vacation time, denoted by τ_{vac}. Now, denoting τ as the cycle time spent by the MC, it is then modelled as in Equation (9):

$$\tau = \tau_p + \tau_{vac} + \tau_c \tag{9}$$

The timing scale of the charging planning should consists of series of charging cycles, charging rounds $(\tau_p + \sum_{k\in Q} \tau_k)$ and charging intervals (τ_{vac}). Charging cycle means that all the cells in the network with SNs have been recharged at least once.

3.2.3 Shortest hamiltonian cycle traveling salesman problem (SHC-TSP): A Hamiltonian path (HP) is a path travelled from a source node to a destination node in a graph, visiting every node en route only once while a *Hamiltonian cycle (HC)* is a closed loop Hamiltonian path, in a graph, where every node (vertex) is visited only once [22]. Therefore, there may be many Hamiltonian cycles for every reference point (source node) in a graph. Thus, length of a graph is the sum of the weights (distances) of its edges (line joining adjacent nodes along the HP) [23]. However, the shortest of these cycles is referred to as the shortest Hamiltonian cycle (SHC), which is of significance to this work.

The shortest Hamiltonian path problem (SHPP) also called Dijkstra's algorithm is similar to traveling salesman problem (TSP) [24]. Although, in SHPP the salesman can start his journey from anywhere, visit every city but do not have to return to starting city, while in TSP, the salesman can also start his journey from anywhere but have to return back to his starting location. Therefore, the SHPP algorithm is a path searching algorithm that iteratively generates all possible routes, from source to destination node, and select the one with the lowest cost (shortest distance) [25, 26].

Hence, in this context, the problem is referred to as Shortest Hamiltonian Cycle Traveling Salesman Problem (SHC-TSP). Note that each cycle depends on the number of requesting SNs in the schedule and each requesting SN belonging to a particular cell send the request alongside its location. Thus, the BS is aware of the location of each requesting SN s_i (x_i, y_i) and its corresponding cell center c_k, (X_i, Y_i) and the location of the MC which it uses to calculate the Euclidian distances along the charging path and use it to plan the recharging schedule. The BS then communicates the schedule to the MC for implementation, as outlined in Table 1.

Table 1: Shortest hamiltonian cycle traveling salesman problem (SHC-TSP) algorithm

Algorithm 1: SHC-TSP

Input: $(X_i,\ Y_i),\ c_0,\ c_k,\ 1 \le k \le |Q|$

Output: Shortest travel path and path length $L\ (m)$

1. Start
2. Select a source cell, denoted as c_0 (service station); % the MC is at liberty to choose to move in clockwise or anti-clockwise direction
3. MC move to a requesting cell c_1 adjacent to c_0; % this determines the cycle direction (clockwise or anti-clockwise)
4. Check the distances from c_1 to its adjacent requesting cells, find the one closer to it along the direction, say c_k and construct the edge (path linking c_k with c_1)
5. Repeat steps 2 to 4 until all the requesting cells in Q are visited
6. MC return to the source cell c_0 after visiting the last cell in the schedule
7. Plot the traveling path $P = \left(c_0, c_{1,} \dots \dots, c_k, \dots \dots c_{|Q|}, c_0\right)$ and return its length $L\ (m)$
8. End.

3.2.4 Simulation: It is assumed that a set of SNs are randomly dispersed over the 2-D network area. Reference to the model in Figure 1, the BS and the SS are located at the cell closest to the origin. The data rate $R_i\ (kbps), i \in \mathbb{N}^+$ is set to be randomly generated and assigned to each SN within the range$[1, 100]$. Let $E_{max} = 100\,J => E_{min} = 5\%\ of\ E_{max} = 0.05 \times 100 = 5\,J$. The simulation was done in MATLAB R2022b environment. Table 2 is the composition of the parameters used.

Table 2: The simulation parameters

Parameter	**Value**
Network area (m^2)	40 x 40
Number of nodes	5
Radius of cells (m)	5
Speed of MC (m/s)	5
Charging radius of MC (m)	5
Initial battery energy of SNs (J)	100
SNs battery energy (SNBE) threshold of (J)	5
Transmit power of the MC (w)	10
MC's antenna gain	3
MC's charging efficiency (C_e)	0.5

Parameter	Value
MC's battery capacity (kJ)	2000
MC moving energy consumption (J/m)	8
SN's antenna gain	2
RF radiation wavelength (λ) (m)	0.0200
Short distance adjustment parameter for Friis equation (δ)	0.2316
Rectifier efficiency (η)	0.9
Antenna polarization loss (L_p)	0.3
Distance between MC and SN ($dist(s_i,\ MC)$) (cm)	10

4. RESULTS AND DISCUSSION

After scripting and running the solution codes in Matlab, several scenarios were observed, but this article captures only two for the purpose of describing the concept, as presented in Tables 3 and 4, and Figures 3 and 4 while Tables 5 and 6 are their corresponding distance matrices. The result shows the shortest path travelled by the MC to visit the centres of the requesting cells, stop and recharge all the nodes within, for two scenarios. In each scenario, the five indexed nodes are randomly dispersed within the network area. The centres of the requesting cells, the travelled path and its length are recorded in each corresponding table of the captured scenarios.

Table 3: Simulation data for scenario 1

Node index	Cell centre (X_i, Y_i)	Traveling path	Path length (m)
0	(3,0)	$c_{\{0\}} \rightarrow c_{\{1,3\}} \rightarrow c_{\{5\}} \rightarrow c_{\{4\}} \rightarrow c_{\{2\}} \rightarrow c_{\{0\}}$	81.9470
1	(3,24)		
2	(28.8,6)		
3	(3,24)		
4	(24.5,9)		
5	(7.3,21)		

Table 4: Simulation data for scenario 2

Node index	Cell centre (X_i, Y_i)	Traveling path	Path length (m)
0	(3,0)	$c_{\{0\}} \rightarrow c_{\{2\}} \rightarrow c_{\{1\}} \rightarrow c_{\{5\}} \rightarrow c_{\{4\}} \rightarrow c_{\{0\}}$	80.0618
1	(11.6,24)		
2	(20.2,6)		
4	(7.3,9)		
5	(3,30)		

Table 5: Distance matrix for scenario 1

Node index	0	1	2	3	4	5
0	0	24	26.4885	24	23.3077	21.4357
1	24	0	31.4585	0	26.2155	5.2431
2	26.4885	31.4585	0	31.4585	5.2431	26.2155
3	24	0	31.4585	0	26.2155	5.2431
4	23.3078	26.2155	9.2431	26.2155	0	20.9724
5	21.4357	5.2431	26.2155	5.2431	20.9724	0

Table 6: Distance matrix for scenario 2

Node index	0	1	2	4	5
0	0	25.9443	18.2165	9.9745	30
1	25.4943	0	19.9489	15.6042	10.4862
2	18.2165	19.9489	0	13.2442	29.5269
4	9.9745	15.6042	13.2442	0	21.4357
5	30	10.4862	29.5269	21.4357	0

Table 7: Comparing energy utility (E_u) between SHC-TSP model and NJNP model

Scenario	SHC-TSP Energy Utility	NJNP Energy Utility	Difference (m)	Savings (%)
1	407.0526	376.3769	30.6757	3.9156
2	333.3099	318.9980	14.3119	2.1940

In the first case presented in Table 3 and Figure 3, all the 5 SNs requested for recharging with 2 out of the 5 (nodes 1 and 3) falling in one cell; thus, the MC charged them simultaneously resulting to a total travelled distance of 81.9470 meters. In the second case, presented in Table 4 and Figure 4, 4 out of the 5 nodes requested for recharging (node 3 did not request) with all of them falling in different cells; thus, the MC had to visit and recharged them individually, covering a distance of 80.0618 meters.

The distance matrices for the two scenarios are calculated based on Equation (2) and presented in Tables 5 and 6, respectively. In scenario 1, based on NJNP, the MC routed through the path $c_{\{0\}} \rightarrow c_{\{5\}} \rightarrow c_{\{1,3\}} \rightarrow c_{\{4\}} \rightarrow c_{\{2\}} \rightarrow c_{\{0\}}$ to cover a longer distance of 88.6259 meters. While in scenario 2, based on NJNP, the MC traversed through the path $c_{\{0\}} \rightarrow c_{\{4\}} \rightarrow c_{\{2\}} \rightarrow c_{\{1\}} \rightarrow c_{\{5\}} \rightarrow c_{\{0\}}$ to cover a distance of 83.6538 meters. Comparing the obtained energy utility simulation data for the two scenarios, as presented in Table 7 and Figure 5, shows that our new SHC-TSP model proves superiority over NJNP model implemented by [19] in terms of minimizing the MC's traveling energy cost with savings of 3.9156% and 2.1940% respectively.

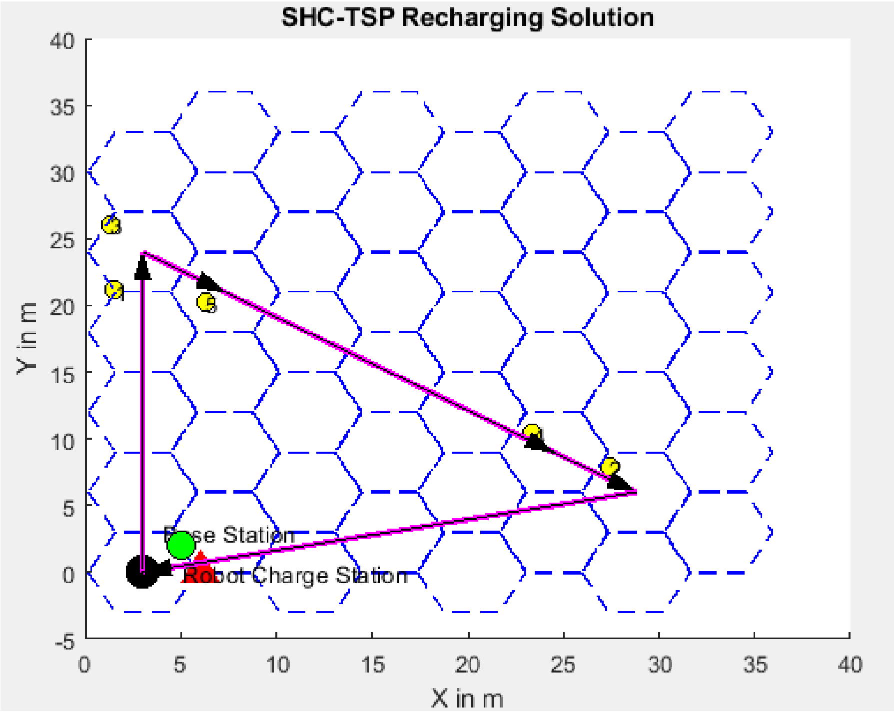

Figure 3: Travelling path for scenario 1

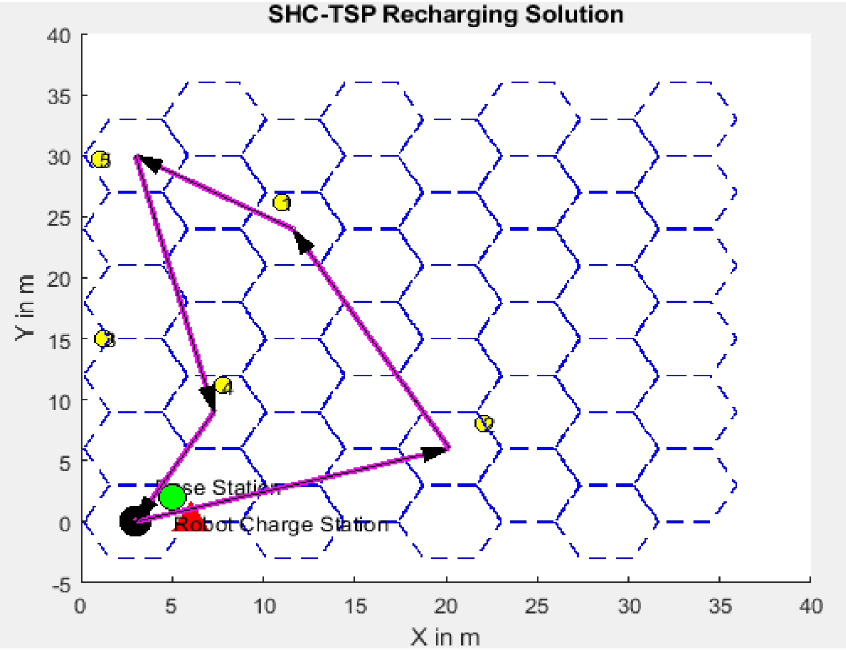

Figure 4: Travelling path for scenario 2

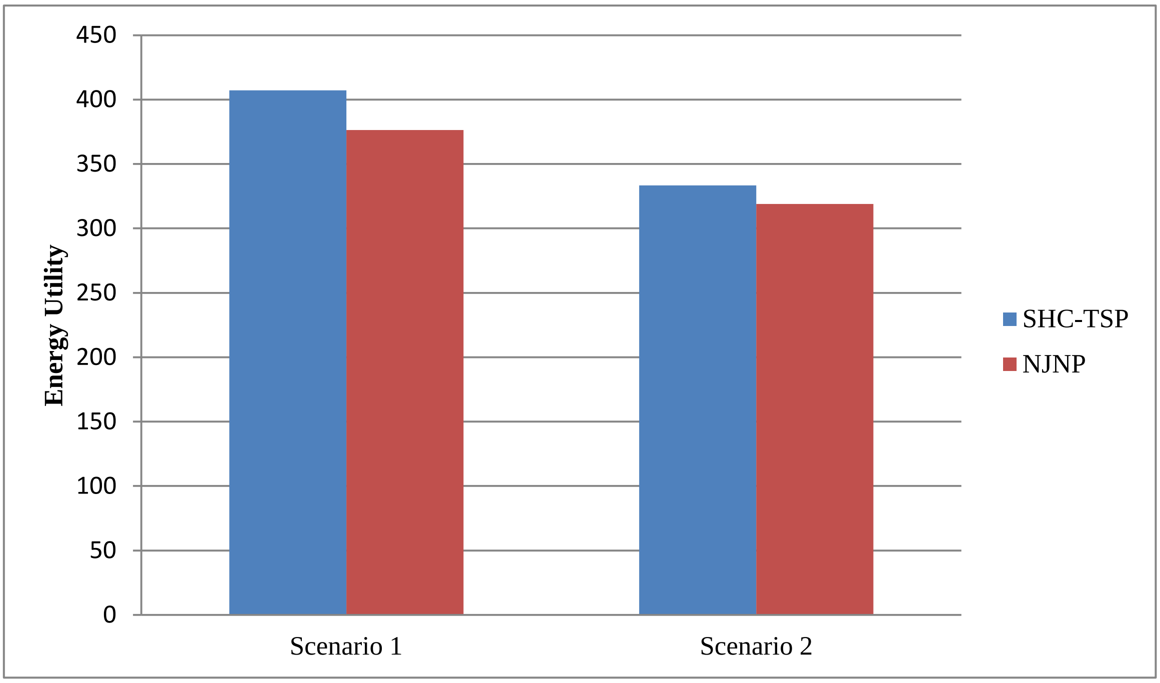

Figure 5: Comparing energy utility: SHC-TSP model versus NJNP model

5. CONCLUSION

Being powered with small batteries of limited capacities and the requirement for continuous operation, makes energy a scarce resource in WSNs whose improvement and efficient utilization is currently a research hub in WSNs domain. The research work planned an optimal charging trajectory able to minimize MC's unnecessary movement energy wastages so as to have enough for meeting the demands of the network SNs based on SHC-TSP, under the limited energy constraint of the MC which can be exhausted at any time, depending on the workload. Comparing the simulation result with that of the notable NJNP model shows that our SHC-TSP model is superior with a success rate of 3.9156% and 2.1940% energy savings for the two analysed scenarios, respectively. Thus, the research delivered a more promising energy-efficient recharging solution for achieving energy sustainability in WRSNs.

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