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Allocation of Power Losses with Distributed Generation using a Contribution of Generator to Load Pairs Technique in a Distribution System

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Abstract: *Nigeria's power sector lacks a structured approach to allocating active power losses, impacting cost fairness and operational efficiency. The contribution of the generator-load pairs technique effectively ensures equitable loss allocation in power networks. This study applied the technique in a Nigerian distribution network integrated with distributed generation (DG) to enhance transparency in loss allocation, using the Imalefalafia 32-bus network as a case study. Power flow analysis was conducted using the backward/forward sweep method to solve the network's power flow problem. The adopted loss allocation technique was then used to allocate active power losses among generators and loads based on their contributions. Results demonstrated accurate and transparent loss allocation, ensuring fairness. The findings confirm the technique's suitability for Nigeria's distribution network and its potential to enhance efficiency in DG-integrated systems. This study introduces an effective loss allocation framework tailored to Nigeria's electricity network, providing a basis for fair active loss-sharing mechanisms.*

Keywords: Distributed generation, loss allocation, distribution network, fairness, accuracy.

1. INTRODUCTION

Distributed generation (DG) is one of the main modern power systems paradigms that is changing highly in both power distribution networks. DG signifies the decentralized electricity generation using small-scale technologies like solar panels, wind turbines and small hydro plants. These technologies are strategically located closer to the point of consumption, which reduces transmission losses, enhances grid reliability, and supports the transition to sustainable energy sources [1, 2]. Across the globe, countries are increasingly incorporating DG into their distribution systems to achieve energy security, reduce carbon emissions, and foster renewable energy adoption.

However, the integration of DG significantly alters the traditional operational dynamics of distribution networks. It introduces complexities such as bidirectional power flow, voltage regulation challenges, and changes in power loss distribution. One critical issue is the equitable allocation of power losses within the network. In deregulated power markets, allocating these losses to various stakeholders—including generators, distribution companies, and consumers—has implications for cost recovery, market fairness, and efficiency [3]. Existing practices, such as absorbing losses by utilities or passing them indirectly to consumers, fail to incentivize efficiency or fairness among market participants.

Nigeria, despite its immense potential for renewable energy integration, has not yet integrated DG into its distribution networks. Current loss allocation practices in the Nigerian electricity market remain rudimentary, often relying on traditional and outdated methods that lack precision. As Nigeria prepares for future DG integration, addressing the challenges of fair and accurate power loss allocation is imperative [4]. Advanced techniques, such as the Contribution of Generator-Load Pairs Technique [5], have been proven effective globally but remain unexplored in the Nigerian context. These techniques provide a mechanism for attributing losses to specific generator-load pairs, ensuring transparency and equity.

This study, therefore, investigates the application of the Contribution of Generator-Load Pairs Technique in power loss allocation for Nigerian distribution systems. Leveraging the Backward/Forward Sweep power flow method, this study offers a robust framework that not only supports equitable loss allocation but also prepares Nigerian networks for the complexities of DG integration. The findings will have significant implications for achieving sustainable, fair, and efficient energy management in Nigeria's evolving power sector.

2. LITERATURE REVIEW

The challenge of loss allocation in distribution networks has attracted significant research attention, particularly with the increasing integration of DG. Various methodologies have been proposed to address this challenge, each offering unique advantages and limitations.

Atanasovski and Taleski [6] introduced the Power Summation Method of Loss Allocation (PSMLA), employing a quadratic scheme for tracing real and reactive power flows. Their approach decomposed power losses into nodal components, providing statistical representations of load and generation curves over time. However, this method primarily focused on theoretical formulations without broad application to practical distribution networks.

Savier and Das [7] compared quadratic and exact methods for loss allocation in radial distribution networks, highlighting the superiority of exact methods in eliminating assumptions inherent in pro-rata schemes. Despite their advantages, these methods did not account for the complexities introduced by DG contributions.

Building on earlier work, Ghofrani-Jahromi et al. [8] proposed a three-step power flow-based method that attributed losses to DG and load nodes equitably. While this method was innovative, it lacked scalability for large systems. Similarly, Jagtap and Khatod [9] employed branch-oriented strategies to directly relate power flow, voltage variations, and losses, addressing nonlinearities in radial distribution networks. Despite their practicality, these methods were not extended to account for deregulated markets or their implications for fair cost distribution.

Sharma and Abhyankar [10] leveraged cooperative game theory and the Shapley value to allocate losses in radial and weakly meshed networks. This approach underscored participant size, network usage, and distance from the source. However, it required complex computations, limiting its adoption in real-time operations. Kashyap and De [11] employed proportional sharing mechanisms, allocating losses based on DG location and size, but their method was less precise for highly meshed systems.

Amaris *et al*. [12] innovatively combined game theory with electrical circuit principles for loss allocation in meshed networks, while Khosravi et al. [13] developed a scalable algorithm considering DG impacts on large systems. Despite these advancements, few studies have explored loss allocation in deregulated environments, particularly for African distribution networks. Chintada *et al*. [14] further analyzed DG placement impacts on loss allocation, emphasizing the need for placement optimization to reduce network losses.

Hata *et al*. [15] tackle the complexity of loss allocation (LA) in distribution networks with high DG penetration and reverse power flows by proposing a circuit theory-based, branch-oriented LA method. This approach simplifies the decomposition of the power loss equation without assumptions, accurately allocating losses based on load demands, power factors, and locations, across various load models and DG types. Validated on 9-bus and 33-bus systems, the method is compared against established techniques and highlights performance under varying load power factors. However, its limitation lies in the assumption of balanced network conditions, which may reduce its practical applicability in real-world systems where unbalanced loads are common.

Koochaki *et al*. [16] present a loss allocation framework based on the power tracking method, aimed at addressing increasing power losses in modern transmission and distribution networks due to rising load demands and the emergence of local energy markets. The proposed method determines how generation is allocated to loads, traces power flows across network lines, and quantifies loss contributions from each transaction between network agents. Its general applicability is demonstrated on a modified IEEE 14-bus system and a 69-bus distribution network. However, the framework assumes ideal conditions without addressing uncertainties such as fluctuations in renewable generation or load variability, which may impact its robustness in real-world dynamic environments.

Atanasovski *et al*. [17] propose a mixed method for transmission loss allocation, combining the Z_{bus} transmission cost allocation and power summation methods. The Z_{bus} method is employed to trace active and reactive power flows and determine each user's contribution, while the power summation method with quadratic allocation handles loss distribution among network nodes. The method is applied to a real transmission model of North Macedonia, characterized by a high share of renewable energy (PV and wind), to support future planning. However, the method assumes a static operating condition, limiting its adaptability to time-varying system behaviors or real-time applications often needed in modern, dynamic power systems.

Elabbas and Camacho [18] introduce a novel transmission loss allocation method that adjusts marginal loss factors (MLFs) to ensure the total allocated losses match actual physical losses, addressing inefficiencies in single-node dispatch models. By formulating the problem as a bilevel optimization—solved via Mathematical

Programming with Equilibrium Constraints (MPEC) in GAMS—the approach minimizes dispatch inefficiency while equitably distributing losses between generators and loads. Applied to a 9-node, 13-generator system, the method improves generator dispatch efficiency and aligns energy prices with true marginal costs. However, the scalability of the approach to large real-world systems remains uncertain, given the computational intensity of solving bilevel MPEC formulations.

Abdelkader *et al*. [19] present a transparent and assumption-free loss allocation method for distribution systems, grounded in circuit theory. The approach fairly allocates losses to power system users and is demonstrated on both radial (28-bus) and meshed (70-bus) networks. The proposed method is compared with three existing techniques and shown to be broadly applicable across different network configurations. However, the method's performance under high DG penetration or dynamic system conditions is not explored, limiting its validation in modern active distribution networks with variable sources and loads.

Singh *et al*. [20] propose a novel loss allocation method for distribution systems with local generators that accounts for consumer power factor variations. Using Kirchhoff's law and a two-stage power flow decomposition, the approach aims to eliminate cross-subsidies and fairly allocate losses based on dynamic active and reactive power flow changes. The method is validated on 30-bus and 69-bus systems and compared favourably with existing techniques. However, the study does not address the impact of rapidly changing load or generation patterns, which may affect accuracy in highly dynamic networks.

Existing studies have primarily focused on theoretical loss allocation frameworks or their application in mature, deregulated markets. However, there is limited research on the practical implementation of loss allocation methods in developing countries, particularly in Nigeria. Advanced techniques like the Contribution of Generator-Load Pairs have been successfully applied in other contexts but remain unexplored in Nigerian distribution networks. Addressing these gaps is essential to develop a fair and transparent mechanism for power loss allocation, especially in anticipation of future DG integration.

This study contributes to the body of knowledge by:

- Introducing the Contribution of Generator-Load Pairs Technique for loss allocation in Nigerian distribution networks, which has not been previously studied.
- Providing a scalable and robust framework using the Backward/Forward Sweep power flow solution, tailored to the specific operational characteristics of Nigerian networks.
- Offering insights into loss allocation mechanisms for deregulated power systems in developing countries, thereby supporting equitable cost-sharing and sustainable energy management.

3. METHODOLOGY

This study focuses on allocating active power losses in a Nigerian distribution network with DG using the Contribution of Generator-Load Pairs Technique. Power flow analysis was conducted using the Backward/forward Sweep power flow method for computational accuracy. The data required, including bus type, power demands, voltage magnitude, and angle, were obtained from the Ibadan Electricity Distribution Company (IBEDC), Nigeria.

3.1 Power Flow Analysis

The backward/forward sweep method was employed for efficient load flow analysis in radial distribution systems. It involves iterative computations, where:

The backward sweep calculates branch currents as given in Equation (1).

$$I_L^{(k)} = I_{\text{load}}^{(k)} + \sum_{\text{child branches}} I_{\text{child}}^{(k)} \quad (1)$$

where $I_{load}^{(k)}$ is the branch current in iteration k.

The forward sweep updates bus voltages as given in Equation (2), where the line impedance was incorporated.

$$V_{\text{bus}}^{(k)} = V_{\text{parent}}^{(k)} - Z_L \cdot I_L^{(k)} \quad (2)$$

where Z_L is the branch impedance.

The process iterates until power mismatches are within acceptable tolerance as given in Equations (3) and (4).

$$\Delta P_k \leq \epsilon \quad (3)$$

$$\Delta Q_k \leq \epsilon \quad (4)$$

where ΔP_k and ΔQ_k are active and reactive power mismatches, respectively.

3.2 Development of Contribution of Generator-Load Pairs Loss Allocation Method

The Generator-Load Pairs Loss Allocation Method ensures equitable allocation of active power losses among market participants. The approach uses current components to calculate losses, leveraging the results of a converged power flow calculation.

1) *Loss Calculation by Current Components:* For a single generator at node 1, the current in branch $l - m$ is the sum of currents of all loads supplied through that branch as given in Equation (5) [5]:

$$I_{l-m} = \sum_{n \in \eta_m} I_n \quad (5)$$

where η_m represents nodes downstream of branch $l - m$.

Following Savier and Das (2009), power losses in any branch are calculated as a product of the branch voltage drop and the conjugate branch current as given in Equation (6).

$$P_{lm} = V_{lm} \cdot I_{lm}^* \quad (6)$$

The total losses from a load at node k are calculated as using Equation (7)

$$P_{loss} = \sum_{\zeta_k} V_{lm} I_{lm}^* \quad (7)$$

Using superposition for multiple generators, the cumulative losses resulting from the load at node k are is calculated using Equation (8) [5], where the current was broken down into its components.

$$P_{loss} = \sum_{\gamma} J_{ki} V_{ik} \quad (8)$$

where J_{ki} is the load current component at node k supplied by generator at node i.

Matrix representations are used to efficiently compute all load current components. The load current component at node k from generator at node i is stored in a matrix J, and voltage differences are stored in a matrix VD. Loss allocation is then calculated using element-wise multiplication using Equation (9).

$$P_{\text{loss,}} = J \odot VD \tag{9}$$

2) *Node and Branch Current Components:* Node currents are broken down into their components as given in Equation (10).

$$I_i = I_{C_i} + I_{L_i} \tag{10}$$

Branch currents are similarly decomposed, and components are stored individually. Real and imaginary parts are calculated using Equations (11) and (12) [5].

$$I'_{BC_{il}} = A' \cdot IC' \tag{11}$$

$$I''_{BC_{il}} = A'' \cdot IC'' \tag{12}$$

where A′ and A″ are transformation matrices derived from power flow results.

3) *Calculation Procedure:* The procedure involves:

(i) Conducting power flow analysis to obtain nodal voltages and branch currents.

(ii) Decomposing node currents into real and imaginary components.

(iii) Using superposition to calculate load current components.

(iv) Allocating losses proportionally among generators and loads using the matrix equations.

This approach ensures accurate, transparent, and fair allocation of active power losses, suitable for DG-integrated systems.

The flowchart for the proposed technique is illustrated in Figure 1.

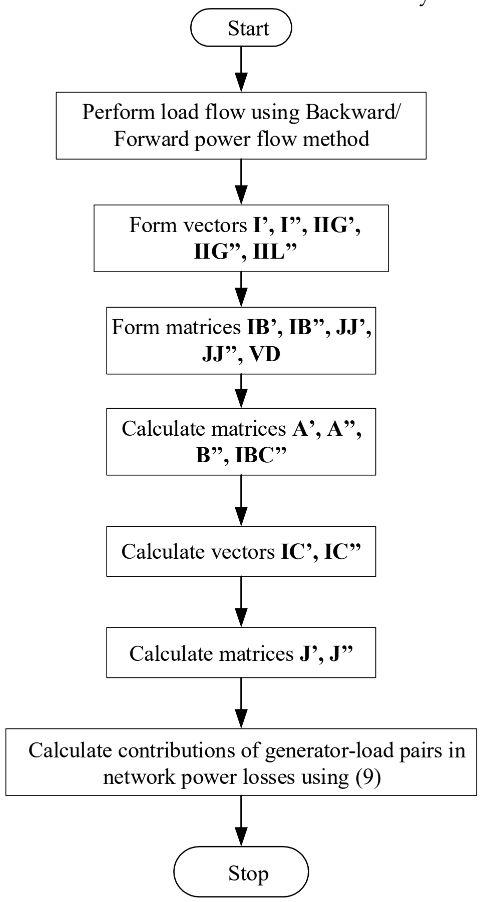

Figure 1: Flowchart of the proposed methodology

3.3 Test Cases

The Imalefalafia 32-bus network, illustrated in Figure 2, comprises 32 buses and 31 branches and forms part of the distribution infrastructure managed by the IBEDC, Nigeria. The network was modelled and simulated using MATLAB/Simulink, a computational tool widely adopted for power system analysis. The network supports total active and reactive power loads of 300 kW and 250 kVAr, respectively. Line and bus data, obtained from IBEDC, are detailed in Table 1.

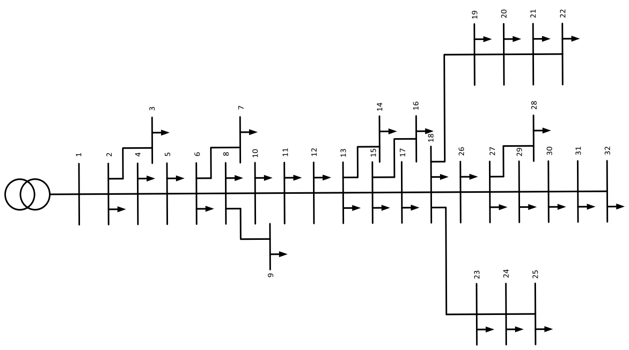

Figure 2: Imalefalafia 32-bus network of IBEDC

Table 1: Bus and line data of Imalefalafia 32-bus Feeder

From Bus	To Bus	R (Ω)	X (Ω)	Load at Receiving End P (kW)	Load at Receiving End Q (kVar)
1	2	0.41975	0.72266	0.00	0.00
2	3	0.07300	0.12568	176.67	57.58
2	4	0.03650	0.06284	141.54	46.21
4	5	0.25550	0.43988	242.15	79.06
5	6	0.01825	0.03142	0.00	0.00
6	7	0.009125	0.01571	195.31	63.76
6	8	0.03650	0.06284	0.00	0.00
8	9	0.009125	0.01571	71.89	23.47
8	10	0.09125	0.15710	204.27	66.69
10	11	0.07300	0.12568	25.87	8.44
11	12	0.01825	0.03142	83.50	27.26
12	13	0.05475	0.09426	0.00	0.00
13	14	0.05475	0.09426	87.17	28.46
13	15	0.03650	0.06284	0.00	0.00
15	16	0.009125	0.01571	25.87	8.44
15	17	0.10950	0.18852	210.38	68.68
17	18	0.21900	0.37704	0.00	0.00
18	27	0.01825	0.03142	57.64	18.82
27	26	0.05475	0.09426	0.00	0.00
26	28	0.01825	0.03142	45.42	14.83
26	29	0.03650	0.06284	117.51	38.36
29	30	0.01825	0.03142	195.11	63.70
30	24	0.05475	0.09426	247.44	80.78
31	32	0.07300	0.12568	229.73	75.00
18	19	0.09125	0.15710	101.22	33.05
19	20	0.18250	0.31420	131.16	42.82
20	21	0.155125	0.26707	133.60	43.62
21	22	0.01825	0.03142	79.43	25.93
18	23	0.01825	0.03142	31.57	10.31
23	24	0.05475	0.09426	104.07	33.98
24	25	0.01825	0.03142	235.43	76.86

4. RESULTS AND DISCUSSION

Figure 3 depicts the voltage profile of the network before the integration of DGs. It reveals a generally satisfactory profile, with buses 18-32 exhibiting voltage magnitudes slightly above the acceptable lower limit of 0.95 p.u., with bus 32 having the lowest voltage magnitude. To enhance network performance and minimize total active power loss, the penetration of DGs is crucial. Figure 4 visually represents total active power losses across all branches, providing insights into the network's energy inefficiency. The simulation results indicate a total active power loss of 70.11 kW, a value that can be significantly reduced through strategic DG placement.

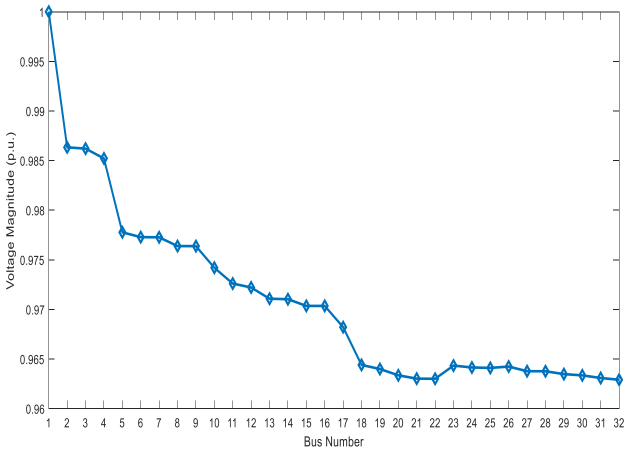

Figure 3: Base case voltage profile

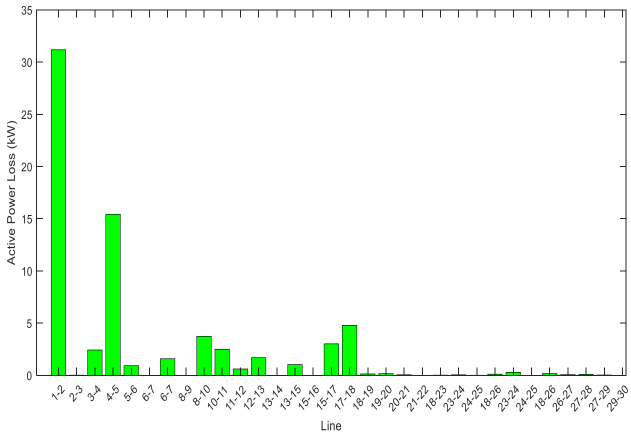

Figure 4: Active power losses in all the lines

4.1 Performance of the Imalefalafia 32-Bus System after Distributed Generation Placement

To provide a comparison, Figure 5 displays the voltage profile before and after the integration of DGs at buses 28–32 in the Imalefalafia 32-bus network. The results show a considerable improvement in the network's voltage profile with the incorporation of DGs.

The voltage magnitudes of buses 18–32, previously slightly above the acceptable lower voltage limit, significantly increased, greatly enhancing the network's performance. Moreover, the total active power loss, which was 70.11 kW, decreased to 29.91 kW, a remarkable 57.34 percent reduction attributed to decreased losses across all branches.

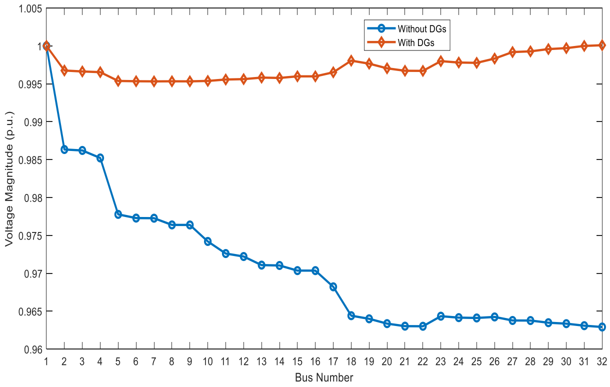

Figure 5: Voltage profile of before and after DG placement

Consequently, the network's efficiency improved, and its reliability was substantially enhanced. Thus, the results indicate that DG penetration has the potential to improve network performance by reducing overall active power loss and enhancing the voltage profile of distribution networks. A more extensive and thorough explanation of the results is provided by the side-by-side comparison of the active power losses in all network lines before and after the integration of DGs, which is shown in Figure 6. Additionally, this illustration may be used to determine how effectively the location of the DG reduces power losses and increases network efficiency.

4.2 Allocation of Active Losses to the Loads on Imalefalafia 32-Bus Network

There are issues with how market participants divide up active losses when DGs are incorporated into radial distribution networks. In order to guarantee equity in the distribution of active power losses among the market participants, the results of implementing the proposed approach to the Imalefalafia 32-bus network with the addition of DG at buses 28 through 32 are presented in this section. Table 2 demonstrates the way this system's loads are allocated to active power loss. As can be seen from Table 2, the loads at buses 28 through 32 had losses that were significantly lower than the loads associated with these buses. This is because some of the loads connected to these buses are receiving power from adjoining generators.

The load at bus 31, with a total of 3.2009 kW, received the greatest share of allotted active power losses, surpassing all other allocated losses. With an allotted loss of 2.7065 kW, the load at bus 32 had the second-largest allocation. After this, allotted losses of 2.2696, 2.2666, 1.9373, 1.7564, 1.6995, and 1.6744 kW were received by loads at buses 30, 25, 17, 5, 29, and 10, respectively. Because of the considerable current these buses drew from the network's generators, the active losses allocated to them were comparatively high. The fact that the network's overall active power loss is determined by the square of the currents passing through every branch has an impact on the amount of active power loss attributed to these buses.

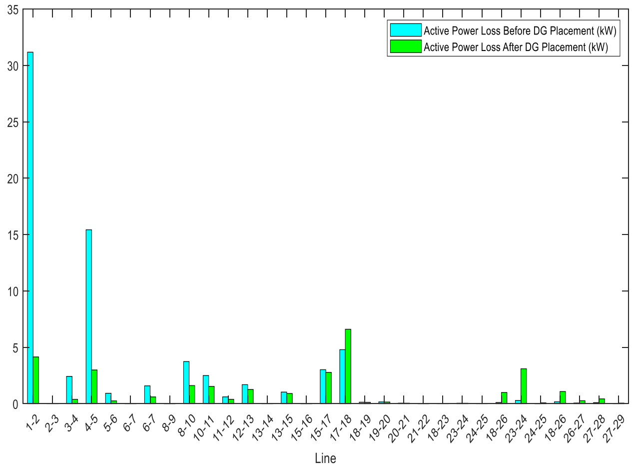

Figure 6: Active power losses across the lines before and after DG placement

Table 2: Active power loss allocation to the loads

Load Bus	Loss Allocated
2	0.0000
3	0.7824
4	0.6707
5	1.7564
6	0.0000
7	1.4477
8	0.0000
9	0.5499
10	1.6744
11	0.2206
12	0.7185
13	0.0000
14	0.7705
15	0.0000
16	0.2306
17	1.9373
18	0.0000
19	0.9827
20	1.3332
21	1.3900
22	0.8271
23	0.2987
24	0.9988
25	2.2666
26	0.5371
27	0.0000
28	1.1258
29	1.6995
30	2.2696
31	3.2009
32	2.7065
Total	**29.91308**

Consequently, these buses made a substantial contribution to the network's overall active power loss because of the large amount of current they drew from the generator. Similarly, buses 2, 6, 8, 13, 15, 18, and 27 were allocated an active power loss of 0 kW to their load. This allocation makes sense because the loads on these buses were zero (0) in magnitude. The results shown in Table 2 illustrate a fair allocation of active power losses among the loads. Based on each load's contribution to the network's overall active power loss, the active power losses were allocated. With this method, every load was given an active power loss that accurately reflected the proportion that it contributed to the network's total active power loss. Overall, the results show that the proposed loss allocation approach is successful in guaranteeing precision and fairness in the distribution of active power losses across loads.

From Table 2, it can be observed that no negative loss allocations exist, which is an expected outcome, as every power exchange between a generator and a load inevitably results in losses on the associated supply line. This result demonstrates the viability of the proposed loss allocation approach and ensures that power losses in the system are appropriately accounted for. Maintaining positive loss allocations is crucial, as negative values would indicate computational errors and provide a misleading representation of the network's performance. Additionally, the total active power allocated matches the total active power of the test network after DG integration, highlighting the accuracy of the proposed approach in allocating active power losses. This highlights the effectiveness, dependability, and robustness of the method as a power loss allocation approach in grids with DGs penetration.

4.3 Allocation of Active Losses to the Generators

Table 3 highlights the simulation results for active loss allocation among the generators. Notably, the generator at bus 1 has the highest allocation, measuring 23.3884 kW. This outcome is unsurprising, as the generator at bus 1 supplies the majority of the network's power. It follows that having a higher active loss allocation than the other generators in the network are normal. Similarly, because it participated in several power transactions throughout the network, the generator at bus 31 had the second-highest active power loss allocation of 1.9822 kW. In comparison to other generators in the network, with the exception of the substation generator at bus 1, this generator participated in power transactions with every lateral in the network and, as a result, made a substantial contribution to the total active power loss in the network. Table 3 offers significant insights into the network's generators' performance and how they affect active power losses.

Significant active power losses of 1.7173, 1.4564, 0.9527, and 0.8985 kW were attributed to generators at busses 28, 29, 30, and 32 during the power transactions between the loads and the generators in the network. This is because the network's overall active power loss was greatly increased by these generators. Due to the relatively small load connected to bus 32, the generator at this bus has the lowest active power loss of 0.8985 kW allocated to it. The load connected to bus 32, as well as a few other buses, was powered by the generator at bus 32. Similar to this, because bus 28's generator supplies neighbouring loads, it was assigned an active power loss of 1.7173 kW. It is important to note that allocating active power losses to generators makes it possible to determine which generators are more responsible for the network's overall active power loss. This information allows for the implementation of strategies to lower losses and enhance the network's overall performance.

Table 3: Active power loss allocation to the generators

To Generator	Loss Allocated
1	23.3884
28	1.7173
29	1.4564
30	0.9527
31	1.9822
32	0.8985
Total	**29.91308**

The total active power loss allocated to the generators in Table 3 was found to be equal to the total loss allocated to the loads in Table 2. This consistency of the total loss in the network was maintained even after the integration of DGs into the system at buses 28 through 32. As a result, this supports the efficacy of the loss allocation method that was proposed in this study, since the total active power loss in the network is the same as the total active power loss allocated to the loads and generators in the network. These results highlight the exactness and reliability of the proposed loss allocation method.

4.4 Power Transactions among the Market Participants on Imalefalafia 32-Bus

A detailed analysis of the power transactions that the market participants in the Imalefalafia 32-bus network engaged in is provided in Table 4. The loads are listed in the leftmost column of the table, and the generators are listed in the first row. Of the loads in the network, generator 1 transacted power with every load aside from buses 2, 6, 8, 13, 15, 18, and buses 27 because no load was connected to these buses and also from buses 28 to 32 due to the fact that generators were connected at these buses, so no power transaction occurred. The largest power transaction occurred between the generator located at bus 1 and the load attached to bus 31, resulting in the allocation of 3.2009 kW of active power between them. On the other hand, the load at bus 11 and generator 1 had the lowest power transactions. This can be clarified by the fact that bus 11 was positioned in between the generators at bus 1 and bus 28. As a result, in their transaction, the load at bus 11 and generator 1 was allocated a small amount of power.

Similarly, all load buses, with the exception of buses 2, 6, 8, 13, 15, 18, and 27, were in power transactions with the generator at buses 28 through 30. Additionally, since the loads connected to buses 28 through 30 were supplied by the generators, there was no power transaction between the generators and the loads. Overall, the simulation results in Table 4 provide compelling evidence that the proposed method is highly effective in allocating active power losses among generators and loads in a radial distribution network.

Notably, the allocation of active power losses appears to be fair among market participants, indicating that the method is highly equitable. Moreover, the proposed method demonstrates a high degree of accuracy, with the total active power allocated to both generators and loads being equal to the total active power loss in the network, as verified in Table 5. These findings suggest that the proposed method has considerable potential for use in power allocation in radial distribution networks.

4.5 Benchmark Validation: Comparative Analysis with IEEE 33-Bus Network

To validate the proposed loss allocation technique, a comparative study was conducted using the IEEE 33-bus network, a standard benchmark for distribution systems, as shown in Table 5. The network's configuration—33 buses, 32 branches, and a total load of 3.72 MW/2.30 MVAr—was augmented with three DGs at buses 6, 25, and 31, aligned with prior studies [15]. The proposed method was compared against the Pro-Rata (PR) method employed by Jagtap and Khatod [9], which allocates losses proportionally based on load/generation magnitudes without accounting for network topology.

Key findings revealed stark contrasts. For instance, at buses 6 and 25—where DGs fully offset local loads—the proposed method allocated 0.00 kW, reflecting localized generation. In contrast, the PR method [9] assigned 0.279 and 0.023 kW to these buses, penalizing DGs for losses they mitigated. At bus 30, a high-loss area, the PR method allocated 21.746 kW, while the proposed method reduced this to 11.6911 kW by tracing losses to upstream generator-load pairs. These results highlight the proposed method's precision in allocating losses equitably, addressing biases inherent in PR [9].

The benchmark analysis strengthens the method's credibility, demonstrating its adaptability to standardized networks like IEEE 33-bus and Nigeria-specific systems like Imalefalafia 32-bus. By resolving gaps in topology-agnostic methods [9], this work aligns with global efforts to incentivize fair DG integration.

Table 4: Power transaction among the market participants

Bus/Gen	1	28	29	30	31	32	**Total**
2	0	0	0	0	0	0	0
3	0.317	0.1119	0.0963	0.0634	0.1332	0.0606	0.7824
4	0.2841	0.0935	0.0801	0.0526	0.1103	0.0501	0.6707
5	0.8018	0.2419	0.1999	0.1295	0.2642	0.1191	1.7564
6	0	0	0	0	0	0	0
7	0.6659	0.1984	0.1638	0.1061	0.2161	0.0974	1.4477
8	0	0	0	0	0	0	0
9	0.2616	0.0732	0.0604	0.0391	0.0797	0.0359	0.5499
10	0.8614	0.2062	0.1703	0.1103	0.2249	0.1013	1.6744
11	0.1218	0.0249	0.0207	0.0134	0.0274	0.0124	0.2206
12	0.4037	0.0794	0.0659	0.0427	0.0874	0.0394	0.7185
13	0	0	0	0	0	0	0
14	0.4537	0.0796	0.0662	0.043	0.0882	0.0398	0.7705
15	0	0	0	0	0	0	0
16	0.1416	0.0222	0.0186	0.0121	0.0249	0.0112	0.2306
17	1.316	0.1519	0.1291	0.0845	0.176	0.0798	1.9373
18	0	0	0	0	0	0	0
19	0.7874	0.0441	0.0399	0.0267	0.058	0.0266	0.9827
20	1.0003	0.0795	0.0688	0.0453	0.0957	0.0436	1.3332
21	1.0083	0.0929	0.0792	0.0519	0.1085	0.0492	1.39
22	0.5991	0.0555	0.0473	0.031	0.0648	0.0294	0.8271
23	0.2482	0.0109	0.0102	0.0069	0.0154	0.0071	0.2987
24	0.8133	0.0411	0.0377	0.0254	0.0557	0.0256	0.9988
25	1.8373	0.0956	0.0874	0.0587	0.1285	0.0591	2.2666
26	0.4636	0.0146	0.0146	0.0101	0.0233	0.0109	0.5371
27	0	0	0	0	0	0	0
28	1.1258	0	0	0	0	0	1.1258
29	1.6995	0	0	0	0	0	1.6995
30	2.2696	0	0	0	0	0	2.2696
31	3.2009	0	0	0	0	0	3.2009
32	2.7065	0	0	0	0	0	2.7065
Total	23.3884	1.7173	1.4564	0.9527	1.9822	0.8985	29.91308

Table 5: Comparison of the proposed method with pro rata method

Proposed Method	Pro Rata Method	Proposed Method
1	0	0
2	0.072	0.0725
3	0.227	0.1282
4	0.511	0.3626
5	0.251	0.1767
6	0.279	0.0000
7	1.591	0.6693
8	2.582	1.6568
9	1.043	0.8317
10	1.387	1.1539
11	1.148	0.8849
12	1.599	1.2821
13	1.915	1.5956
14	4.045	3.3355
15	2.012	1.8650
16	2.112	1.8947
17	2.210	1.9925
18	3.373	2.9758
19	0.110	0.1006
20	0.377	0.3769
21	0.427	0.4272
22	0.471	0.4707
23	0.264	0.1849
24	1.034	1.1155
25	0.023	0.0000
26	0.424	0.1841
27	0.797	0.3066
28	1.211	0.8048
29	3.417	2.7991
30	21.746	11.6911
31	4.675	1.4513
32	6.601	1.8837
33	2.297	0.7552

5. CONCLUSION

This study demonstrates the effectiveness of the Contribution of Generator-Load Pairs Technique for allocating active power losses in radial distribution networks with DG. By leveraging the Backward/Forward Sweep method and superposition principles, the approach ensures fairness and transparency in loss allocation, as validated through the Imalefalafia 32-bus and IEEE 33-bus networks. The integration of DGs reduced total active power losses by 57.34% in the Imalefalafia network, while comparative analysis with the Pro-Rata method highlighted the proposed technique's superior accuracy in attributing losses to generator-load interactions. These findings position the method as a robust framework for Nigeria's evolving power sector and deregulated markets.

However, the study has limitations that warrant future exploration. The current framework assumes steady-state conditions and radial network topologies. Its performance in weakly meshed networks—common in urban grids with looped configurations—remains untested, as meshed systems may introduce complexities such as bidirectional power flows that challenge the superposition-based allocation logic. Additionally, the method's scalability to larger systems (e.g., 100+ buses) and computational efficiency for real-time applications require further investigation. Dynamic scenarios involving intermittent renewable generation or fluctuating loads also lie beyond the scope of this work. Future research will extend the method to address these challenges, incorporating probabilistic load flow models and adaptive algorithms to enhance its applicability in diverse operational contexts.

Despite these limitations, the proposed technique offers a critical foundation for equitable loss allocation in DG-integrated systems. By resolving biases inherent in traditional methods, it supports Nigeria's transition to decentralized, sustainable energy systems while fostering transparency among market participants.

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